EXAMPLE 11 - CAST-IN-PLACE CONCRETE CANTILEVER RETAINING WALL

GENERAL INFORMATION

Example 11 demonstrates design procedures for cast-in-place cantilever retaining walls supported on spread footing in conformance with AASHTO and Section 11.5 of this BDM. Horizontal earth pressure is applied based on the Coulomb earth pressure theory.

Example Statement: The retaining wall supports 15'-0" of level roadway embankment measured from top of wall to top of footing. The wall will be built adjacent to the roadway shoulder where traffic is 2 ft. from the barrier face. The wall stem is 1'-6" wide to accommodate mounting a Type 7 Bridge Rail to the top of wall. See Figure 3.

Starting Element Size Assumptions:

Total Footing Width = 70% to 75% of the design height

Footing Thickness = 10% of the design height

Toe Width = 10% of design height

MATERIAL PROPERTIES

Soil: CDOT Class 1 Backfill-Drained

Footing bears on soil				
Soil unit weight	γ_s =	0.130	kcf	
Angle of internal friction (backfill)	ϕ =	34	deg	
Wall-backfill friction angle	$\delta = 2/3\phi$ =	22.67	deg	
Coefficient of active earth pressure	K_a =	0.261	(Coulomb)	AASHTO Eq. 3.11.5.3-1
Coefficient of passive earth pressure	K_p =	7.60		AASHTO Fig. 3.11.5.4-1
Active equivalent fluid weight	EFW (a) = $K_a\ \gamma_s$ =	0.036	kcf (36 pcf min)	BDM 11.5
Passive equivalent fluid weight	EFW (p) = $K_p\ \gamma_s$ =	0.988	kcf	

Subgrade: for bearing and sliding

Nominal design values are typically provided in the project-specific geotechnical report.

Nominal soil bearing resistance	q_n =	7.50	ksf	
Angle of internal friction (subgrade)	ϕ_{Sub} =	20	deg (for sliding)	
Wall-subgrade friction angle	$\delta_{Sub} = 2/3\phi_{Sub}$ =	13.33	deg (for shear key design)	
Nominal soil sliding coefficient	$\mu_n = \tan\phi_{Sub}$ =	0.36		AASHTO C.10.6.3.4

Concrete: CDOT Concrete Class D

Concrete compressive strength	f'c =	4.50	ksi
Concrete unit weight	γ_c =	0.150	kcf

Bridge Rail Type 7

Type 7 bridge rail weight	w_{rail} =	0.486	klf
Center of gravity from wall back face	$X_{C.G.}$ =	6.84	in. (see Bridge Worksheet B-606-7A)

RESISTANCE FACTORS

When not provided in the project-specific geotechnical report, refer to the indicated AASHTO sections.

Bearing	ϕ_b=	0.55		AASHTO T.11.5.7-1
Sliding (concrete on soil)	ϕ_T=	1.00		AASHTO T.11.5.7-1
Sliding (soil on soil)	$\phi_{T\ s\text{-}s}$=	1.00		AASHTO T.11.5.7-1
Passive pressure	ϕ_{ep}=	0.50		AASHTO T.10.5.5.2.2-1
Extreme event	ϕ_{EE}=	1.00		AASHTO 11.5.8

WALL GEOMETRY INFORMATION

See Figure 1.

Stem Height	H =	15.00	ft.		
Top of Wall Thickness	T_{Top} =	1.50	ft.		
Bottom of Wall Thickness	T_{Bot} =	1.75	ft.		
Width of footing	B =	10.00	ft.		
Thickness of Footing	T_F =	1.25	ft.		
Toe Distance	S =	2.75	ft.		
Height of fill over the toe	H_{TF} =	2.00	ft.		BDM 11.5.1
Minimum Footing embedment ≥ 3 ft..	H_{TF} + T_F =	3.25	ft.	**OK**	BDM 11.5.1
Bridge Rail Type 7 Height	H_B =	2.92	ft.		
Wall Backface to vertical surcharge	R =	2.00	ft.		
Live Load Surcharge height	h_{Sur} =	2.00	ft.		AASHTO Table 3.11.6.4-2
Vehicle Collision Load (TL-4)	P_{CT} =	54.00	kip		AASHTO Table A13.2-1
Collision Load Distribution	L_t =	3.50	ft.		AASHTO Table A13.2-1
Top of wall to point of collision impact on rail	h_{CT} =	2.67	ft.		

1. STABILITY CHECKS

Use the load combinations and factors from AASHTO 11.5.6 and BDM Section 11.5.1 for all loads acting on the retaining wall. Evaluate the retaining wall for the following:

1. Eccentricity
2. Sliding
3. Bearing

Note: The Geotechnical Engineer is responsible for evaluating global stability with consideration for both footing width and embedment.

APPLIED LOADS

Loads not listed here may be applicable for different design cases.

DC - dead load of structural components and nonstructural attachments

EH - horizontal earth pressure load

EV - vertical pressure from dead load of earth fill

CT - vehicular collision force

LS - live load surcharge

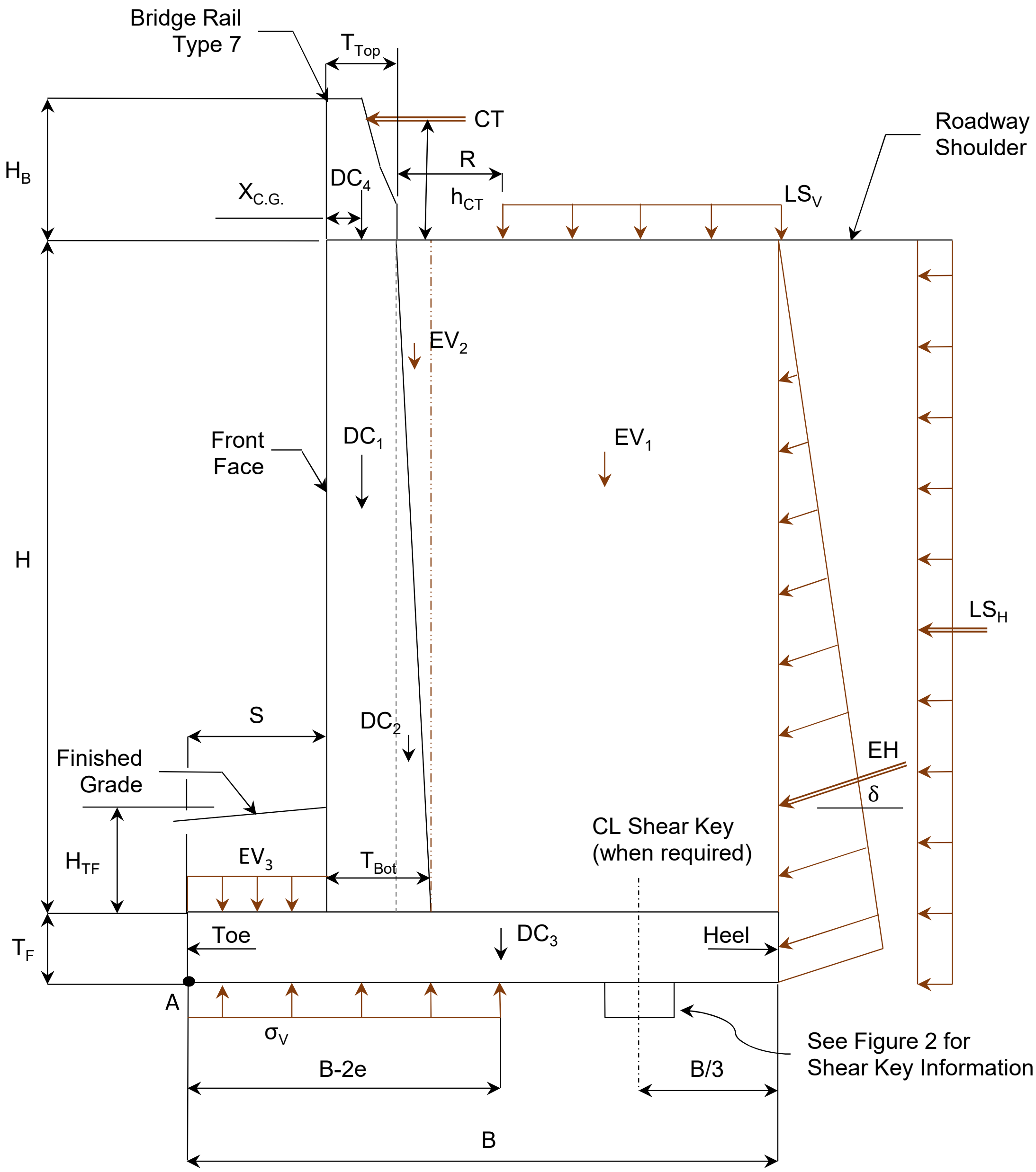

Figure 1 - Typical Section

Summary of Unfactored Loads and Moments

Resolve moments about Point A (see Figure 1 - Typical Section)

Vertical Loads & Moments				
Load Type	Description	V (kip/ft.)	Moment Arm (ft.)	MV (kip-ft.)/ft.
DC_1	Stem dead load	3.38	3.50	11.83
DC_2	Stem dead load	0.28	4.33	1.21
DC_3	Footing dead load	1.88	5.00	9.40
DC_4	Barrier dead load	0.49	3.32	1.63
EV_1	Vertical pressure from dead load of fill on heel	10.73	7.25	77.79
EV_2	Vertical pressure from dead load of fill on heel	0.24	4.42	1.06
EV_3	Vertical pressure from dead load of fill on toe	0.72	1.38	0.99
EH_V	Vertical component of horizontal earth pressure	1.83	10.00	18.30
LS_V	Vertical component of live load surcharge	0.98	8.13	7.97

Horizontal Loads and Moments				
Load Type	Description	H (kip/ft.)	Moment Arm (ft.)	MH (kip-ft.)/ft.
EH_H	Horizontal component of horizontal earth pressure	4.39	5.42	23.79
LS_H	Horizontal component of live load surcharge	1.17	8.13	9.51
CT	Vehicular collision load	2.61	18.92	49.38

$$EH_V = \sin(\delta)\, EH = \sin(\delta)\, 0.5\, EFW(a)\, (H + T_F)^2$$
$$EH_H = \cos(\delta)\, EH = \cos(\delta)\, 0.5\, EFW(a)\, (H + T_F)^2$$
$$LS_V = \gamma_s\, h_{Sur}\, (B - S\ - T_{Top} - \mathrm{R})$$
$$LS_H = EFW(a) h_{Sur}\, (H + T_F)$$

Note: The collision force (CT) is assumed to be distributed over a length of “Lt” ft. at the point of impact and is also assumed to spread downward to the bottom of the footing at a 45° angle. Conservatively, CT is assumed at the end of the wall where the force distribution occurs in one direction. See Figure 11-20 in Section 11 of this BDM.

Reinforcement between the Bridge Rail Type 7 and the wall interface is assumed to be adequate to transfer the collision load from the rail through the wall to the footing.

$$CT = P_{CT} / \left(L_t/2 + \ (h_{CT} + H + T_F) \right)$$

Load Combinations

The table that follows summarizes the load combinations used for the stability and bearing checks of the wall. To check sliding and eccentricity, load combinations Strength Ia and Extreme Event IIa apply minimum load factors to the vertical loads and maximum load factors to the horizontal loads. To check bearing, load combinations Strength Ib, Strength IV, and Extreme Event IIb apply maximum load factors for both vertical and horizontal loads.

CT load is considered with Extreme Event II limit state when checking eccentricity, sliding, and bearing.

Note: LS_H, LS_V, and EH_H are not included in Extreme Event IIa or IIb. It is assumed that the horizontal earth pressure is not activated due to the force of the collision deflecting the wall away from the soil mass at the instant of collision.

LS_V is not applied when analyzing sliding and overturning; rather, it is applied only for load combinations that are used to analyze bearing (AASHTO 11.5.6, Figure C11.5.6-3a).

The service limit state is used for the crack control check and settlement.

Total factored force effect: $Q = \Sigma\, \eta_i \gamma_i Q_i$ AASHTO 3.4.1-1

where Q_i = force effects from loads calculated above

Load Modifiers:	Ductility	η_D =	1.00	AASHTO 1.3.3-1.3.5
	Redundancy	η_r =	1.00	
	Operational Importance	η_I =	1.00	

Load Factors:

Load Combination	γ_{DC}	γ_{EV}	γ_{LS_V}	γ_{LS_H}	γ_{EH}	γ_{CT}	Application
Strength Ia	0.90	1.00	-	1.75	1.50	-	Sliding, Eccentricity
Strength Ib	1.25	1.35	1.75	1.75	1.50	-	Bearing, Strength Design
Strength IV	1.50	1.35	-	-	1.50	-	Bearing
Extreme IIa	0.90	1.00	-	-	-	1.00	Sliding, Eccentricity
Extreme IIb	1.25	1.35	-	-	-	1.00	Bearing
Service I	1.00	1.00	1.00	1.00	1.00	-	Wall Crack Control

Summary of Load Groups:

Load Combination	Vertical Load & Moment		Horizontal Load & Moment	
	V (kip/ft.)	MV (kip-ft.)/ft.	H (kip/ft.)	MH (kip-ft.)/ft.
Strength Ia	19.86	128.95	8.63	52.33
Strength Ib	27.78	179.27	8.63	52.33
Strength IV	27.57	171.34	6.59	35.69
Extreme IIa	17.12	101.50	2.61	49.38
Extreme IIb	23.32	137.87	2.61	49.38
Service I	20.53	130.18	5.56	33.30

Eccentricity (Overturning) Check

When a shear key is required to prevent sliding, the passive resistance shall be ignored.

Maximum eccentricity limit: e_{max} = B/3 = 3.33 ft. AASHTO 10.6.3.3

$$e_{actual} = \frac{B}{2} - \frac{\Sigma M_V - \Sigma M_H}{\Sigma V}$$

Strength Ia:	X =	$(\Sigma M_V - \Sigma M_H) / \Sigma V$ =	(128.95 - 52.33) / 19.86 =	3.86	ft.		
	e =	10.0 / 2 - 3.86 =	1.14 ft.	e_{actual}	<	e_{max}	**OK**
Extreme IIa:	X =	$(\Sigma M_V - \Sigma M_H) / \Sigma V$ =	(101.50 - 49.38) / 17.12 =	3.04	ft.		
	e =	10.0 / 2 - 3.04 =	1.96 ft.	e_{actual}	<	e_{max}	**OK**

Bearing Resistance Check

When a shear key is required to prevent sliding, the passive resistance shall be ignored.

Vertical stress for wall supported on soil: $\sigma_v = \frac{\Sigma V}{B - 2e}$ AASHTO 11.6.3.2-1

Nominal soil bearing resistance	q_n =	7.50	ksf	
Factored bearing resistance	$q_R = \phi_b\, q_n$ =	4.13	ksf	
	$q_{R\_EE} = \phi_{EE}\, q_n$ =	7.50	ksf	Extreme event

Strength Ib:	X =	$(\Sigma M_V - \Sigma M_H) / \Sigma V$ =	(179.27 - 52.33) / 27.78 =	4.57	ft.	
	e =	B / 2 - X =	10.0 / 2 - 4.57 =	0.43	ft.	
	σ_V =	ΣV / (B-2e) =	27.78 / (10.0 - 2 (0.43)) =	3.04	ksf	
				σ_V <	q_R	**OK**
Strength IV:	X =	$(\Sigma M_V - \Sigma M_H) / \Sigma V$ =	(171.34 - 35.69) / 27.57 =	4.92	ft.	
	e =	B / 2 - X =	10.0 / 2 - 4.92 =	0.08	ft.	
	σ_V =	ΣV / (B-2e) =	27.57 / (10.0 - 2 (0.08)) =	2.80	ksf	
				σ_V <	q_R	**OK**
Extreme IIb:	X =	$(\Sigma M_V - \Sigma M_H) / \Sigma V$ =	(137.87 - 49.38) / 23.32 =	3.79	ft.	
	e =	B / 2 - X =	10.0 / 2 - 3.79 =	1.21	ft.	
	σ_V =	ΣV / (B-2e) =	23.32 / (10.0 - 2 (1.21)) =	3.08	ksf	
				σ_V <	q_{R_EE}	**OK**

Sliding Check AASHTO 10.6.3.4

Per AASHTO 11.6.3.5, passive soil pressure shall be neglected.

Strength Ia and Extreme IIa:

Maximum total Horizontal force	ΣH =	8.63	kip / ft.	
Maximum total Vertical force	ΣV =	19.86	kip / ft.	
Nominal passive resistance	R_{ep} =	0.00	kip / ft.	AASHTO 11.6.3.5
For concrete cast against soil	C =	1.00		AASHTO EQ 10.6.3.4-2
Nominal soil sliding coefficient	$\mu_n = \tan \phi_{Sub}$ =	0.360		
Nominal sliding resistance	$R_\tau = C\ \Sigma V \mu_n$ =	1.0 (19.86) (0.360) =	7.15	kip / ft.

Factored resistance against failure by sliding

$$R_R = \phi R_n = \phi_\tau R_\tau + \phi_{ep} R_{ep} = \quad 1.00\ (7.15) + 0.50\ (0.0) = \quad 7.15 \quad \text{kip / ft.}$$

R_R < ΣH **Shear Key is Required**

Shear Key Design

1. Assume shear key dimensions.
2. Center line of the shear key is approximately B/3 from the heel edge of the footing; see BDM Section 11.5.1.
3. Passive soil pressure at the toe shall be neglected; only include passive pressure due to the inert block (c) (see AASHTO 11.6.3.5).
4. Depth of inert block is taken to be the sum of the key depth and the effective wedge depth. This example follows this methodology. Conservatively, effective wedge depth can be ignored, allowing inert block to be equal to shear key depth.
5. Per BDM Section 11.5.1, the top 1 ft. of fill at the toe shall be ignored for all design cases.
6. The Designer may choose to add weight of the shear key for eccentricity and bearing analysis once shear key dimensions are confirmed. For this example, weight of the key is ignored.

Shear key depth	d_{Key} =	1.00	ft.
Shear key width	T_{Key} =	1.50	ft.
Heel of footing to centerline shear key	K =	3.50	ft.
Toe of footing to front face of shear key	X_{Key} =	5.75	ft.
Soil cover above the footing toe	H_{TF} =	2.00	ft.
Shear friction angle of subgrade	$\delta_{sub} = 2/3\varphi_{sub}$ =	13.33	deg.
Inert block depth	$c = d_{Key} + X_{Key}\tan(\delta_{sub})$ =	2.36	ft.
Top of fill to top of shear key	y_1 =	2.25	ft.
Top of fill to bottom of inert block	y_2 =	4.61	ft.

Passive equivalent pressure	EFW (p) =	0.988	kcf	
Nominal soil sliding coefficient	μ_n =	0.360		
Coefficients of friction (factored):	$\mu_u = \phi_T\ \mu_n$ =	1.00 (0.360) =	0.360	(concrete-soil)
	$\mu_{u\ s\text{-}s} = \phi_{T\ s\text{-}s}\ \mu_n$ =	1.00 (0.360) =	0.360	(soil-soil)
	$\mu_{u\ EE} = \phi_{EE}\ \mu_n$ =	1.00 (0.360) =	0.360	(extreme event)

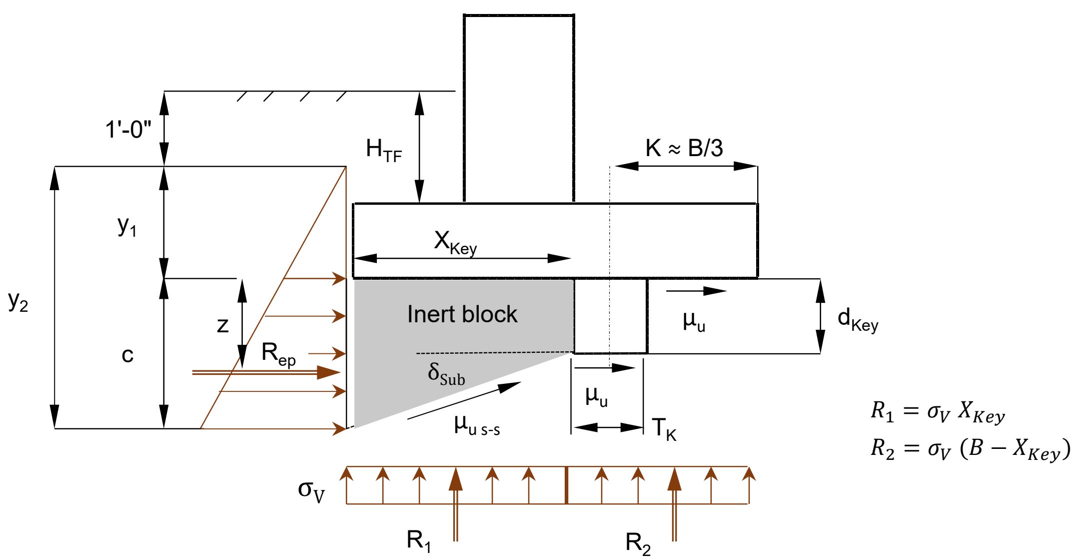

Figure 2 - Shear Key

Shear resistance between soil and foundation:

$$\phi_\tau R_\tau = C\ R_1\ \mu_{u\ s-s} \cos\delta_{Sub} + C\ R_2\ \mu_u \quad \text{(Strength Ia)}$$

$$\phi_{EE} R_\tau = C\ R_1\ \mu_{u\ EE} \cos\delta_{Sub} + C\ R_2\ \mu_{u\ EE} \quad \text{(Extreme IIa)}$$

$$X = (\Sigma M_V - \Sigma M_H)/\Sigma V \qquad e = \frac{B}{2} - X \qquad \sigma_v = \frac{\Sigma V}{B - 2e}$$

Load Combination	ΣV (kip/ft.)	Σ MV (kip-ft./ft.)	Σ MH (kip-ft./ft.)	X (ft.)	e (ft.)	σ_V (ksf)	R1 (kip/ft.)	R2 (kip/ft.)	ϕRτ (kip/ft.)
Strength Ia	19.86	128.95	52.33	3.86	1.14	2.57	11.42	8.44	7.04
Extreme IIa	17.12	101.50	49.38	3.04	1.96	2.82	9.84	7.28	6.07

Passive resistance of soil available throughout the design life of structure:

$R_{ep} = EFW(p) 0.5\ (y_1 + y_2)\ c =$ 0.988 * 0.5 (2.25 + 4.61) 2.36 = 8.00 kip

Factored resistance against failure by sliding: AASHTO 10.6.3.4

Strength Ia: Maximum total Horizontal force ΣH = 8.63 kip

$R_R = \varphi R_n = \varphi_\tau R_\tau + \varphi_{ep} R_{ep} =$ 7.04 + 0.50 (8.00) = 11.04 kip

R_R > ΣH **OK**

Extreme IIa: Maximum total Horizontal force ΣH = 2.61 kip

$R_R = \varphi R_n = \varphi_{EE} R_\tau + \varphi_{ep} R_{ep}$ = 6.07 + 0.50 (8.00) = 10.07 kip

R_R > ΣH **OK**

2. STRENGTH DESIGN

Concrete compressive strength	f'_C =	4.50	ksi	
Yield strength of the reinforcement	fy =	60.00	ksi	
Concrete unit weight	γ_c =	0.150	kcf	
Correction factor for source aggregate	K_1 =	1.00		AASHTO 5.4.2.4
Modulus of elasticity of reinforcement	E_S =	29000	ksi	AASHTO 5.4.3.2
Modulus of elasticity of concrete	$E_C = 120{,}000 K_1 \gamma_c^2 f_c'^{0.33} =$	4435.31	ksi	AASHTO 5.4.2.4
Modular ratio	n = E_S / E_C =	6.54		AASHTO 5.6.1
Compression zone factor	$\beta 1 = 0.85 - (f'c - 4.0) 0.05 =$	0.825		AASHTO 5.6.2.2
Resistance factor for flexural-tension control	ϕ_f =	0.90		AASHTO 5.5.4.2
Resistance factor for shear-tension control	ϕ_v =	0.90		AASHTO 5.5.4.2
Design width	b=	12.00	in.	

2.1 STEM WALL DESIGN

Summary of Unfactored Horizontal Loads and Moments at the Bottom of the Stem:

Load Type	Description	H (kip/ft.)	Moment Arm (ft.)	MH (kip-ft.)/ft.
EH_H	Soil	3.74	5.00	18.70
LS_H	Surcharge	1.08	7.50	8.10

Summary of Load Groups:

Load Combination	Horizontal Load & Moment	
	Vu (kip/ft.)	Mu (kip-ft.)/ft.
Strength Ib	7.50	42.23
Service I	4.82	26.80

It has been assumed that the load combination Strength Ib generates the maximum moment at the interface of the stem wall and footing. However, the Designer should check all possible load combinations, including extreme event, and select the combination that produces the maximum load for the design of the stem.

Note: The Designer/Engineer is encouraged to use engineering judgment to determine the moment and required area of reinforcing steel at other points of the stem for tall walls (H ≥ 10.0') to reduce the amount of steel required at higher elevations.

2.1.1 Flexure Design

AASHTO 5.6.3.2

Design of vertical reinforcement bars at back face of stem

Assumed bar size	Bar =	# 5	
Factored applied moment	$M_{u\ Str}$ =	42.23	kip-ft. / ft.
Concrete clear cover	r =	2.00	in.
Bar diameter	d_b =	0.625	in.
Bar area	A_b =	0.310	in^2

Effective Depth $d_e = T_{Bot} - r - d_b / 2 =$ 1.75' (12) - 2" - 0.625" / 2 = 18.69 in.

Try # 5 @ 6.0" on center:

Design steel area	$A_S = A_b\, b / spa =$	0.310 (12) / 6 =	0.620	in^2/ft.
Distance from compression fiber to neutral axis	$C_b = \dfrac{A_s f_y}{\beta_1 0.85 f_c' b} =$	0.620 (60) / (0.825*0.85*4.5*12) =	0.982	in.
Equivalent Stress Block	$a = \beta_1 C_b =$	0.825 (0.982) =	0.810	in.
Nominal Flexural Resistance	$M_n = A_S f_y \left(d_e - \dfrac{a}{2}\right) =$	0.620 (60) (18.69 - 0.810 / 2) =	56.68	kip-ft.
Factored Flexural Resistance	$M_R = \phi_f M_n =$	0.90 (56.68) =	51.01	kip-ft.

M_R > $M_{u\ Str}$ **OK**

<u>Maximum Reinforcement:</u> Provision deleted in 2005

<u>Minimum Reinforcement:</u>

AASHTO 5.6.3.3

The amount of tensile reinforcement shall be adequate to develop a factored flexural resistance, M_R, at least equal to the lesser of $1.33M_{u\ Str}$ or M_{cr}

Member width	b =	12.00	in.
Member depth	$d = T_{Bot}$ =	21.00	in.
Distance to Neutral Axis	$y_t = T_{Bot} / 2$ =	10.50	in.
Stem moment of inertia	$I_g = b\, d^3 / 12$ =	9261.0	in^4
Section modulus	$S_{nc} = S_c = I_g / y_t$ =	882.0	in^3

Concrete Modulus of Rupture $f_r = 0.24\sqrt{f_c'} =$ 0.509 ksi AASHTO 5.4.2.6

Cracking moment, $M_{cr} = y_3\left[\left(y_1 f_r + y_2 f_{cpe}\right)S_c - M_{dnc}\left(\gamma_c/\gamma_{nc} - 1\right)\right]$: AASHTO 5.6.3.3-1

Flexural cracking variability factor	$y_1 =$	1.600		
Prestress variable factor	$y_2 =$	0.000		
Ratio of specified minimum yield strength to ultimate tensile strength of the reinforcement	$y_3 =$	0.670	for A615, Grade 60 steel	
Compressive stress due to prestress force	$f_{cpe} =$	0.000	ksi	
Total unfactored dead load moment	$M_{dnc} =$	0.000	kip-in.	
Cracking moment,				
$M_{cr} =$	0.670 [(1.60 * 0.509 + 0) * 882.0 - 0] / 12 =	40.11	kip-ft./ft.	- controls
Factored applied moment *1.33	$1.33\ M_{u\ Str} =$	56.17	kip-ft./ft.	
Factored flexural resistance	$M_R =$	51.01	kip-ft./ft.	

M_R > min (Mcr, $1.33M_{u\ Str}$) **OK**

<u>Control of cracking by distribution of reinforcement:</u> AASHTO 5.6.7

Exposure condition class	2	Use Class 2 for the stem, Class 1 for the footing and key		
Exposure factor	$\gamma_e =$	0.75		
Thickness of concrete cover	d_c = 2" + d_b / 2 =	2" + 0.625 / 2 =	2.31	in.
Reinforcement Ratio	$\rho = A_S/bd_e =$	0.620 / (12 * 18.69) =	0.003	
Modular ratio	n =	6.54		
	$k = \sqrt{2n\rho + (n\rho)^2} - n\rho =$	0.179		
	$j = 1 - k/3 =$	0.940		
Service applied moment	$M_{u\ serv} =$	26.80	kip-ft.	
Tensile stress in steel	$f_{ss} = M_{u\ serv} * 12/(A_S j d_e) =$	26.80(12) / (0.620*0.940*18.69) =	29.52	ksi
	$\beta_s = 1 + \dfrac{d_c}{0.7(T_{Bot} - d_c)} =$	1 + 2.31 / 0.7 (1.75*12 - 2.31) =	1.18	
Maximum spacing	$s_{max} = \dfrac{700\gamma_e}{\beta_s f_{ss}} - 2d_c =$	700 (0.75) / (1.18*29.52) - 2(2.31) =	10.45	in.
Spacing provided	$s_{prov} =$	6.00	in.	

s_{prov} < s_{max} **OK**

2.1.2 Shear Design AASHTO 5.7.3.3

Shear typically does not govern the design of retaining walls. If shear becomes an issue, the thickness of the stem should be increased. Ignore benefits of the shear key (if applicable) and axial compression.

Factored shear load	$V_{u\ str} =$	7.50	kip/ft.	
Effective Depth	d_V = max (d_e - C_b/2, *0.9 d_e, 0.72 T_{Bot}*) =	18.20	in (shear)	AASHTO 5.7.2.8

Per AASHTO 5.7.3.4.1, this section does not qualify for simplified procedure for determining shear resistance parameters. General procedure will be used (AASHTO 5.7.3.4.2).

Longitudinal tensile strain in the section

$$\varepsilon_s = \frac{\left(\dfrac{|M_u|}{d_v} + 0.5N_u + \left|V_u - V_p\right| - A_{ps}f_{po}\right)}{E_s A_s + E_p A_{ps}}$$

Removing all prestress steel unknowns, the equation will be as follows:

$$\varepsilon_s = \frac{\left(\dfrac{|M_u|}{d_v} + 0.5N_u + |V_u|\right)}{E_s A_s}$$

$E_s A_s$

Where,

Factored moment $M_u = \max(M_{u\,str}, V_{u\,str} * d_v) =$ 42.23 kip-ft./ft.

Factored axial force $N_u = 1.25\,(DC_1+DC_2+DC_4) =$ -5.19 kip

Area of steel on the flexural tension side $A_s =$ 0.620 in^2 / ft.

Modulus of elasticity of reinforcement $E_s =$ 29,000 ksi

Longitudinal tensile strain in the section $\varepsilon_s =$ 0.00182 in / in

Parameter β for sections with no transverse reinforcement $$\beta = \frac{4.8}{(1+750\varepsilon_s)}\frac{51}{(39+s_{xe})}$$

Where,

Crack spacing parameter (1) $s_x = \min \begin{cases} d_v = 18.20 \text{ in} \\ s = 12.00 \text{ in (see below - \#4 @ 12")} \\ \text{if } A_{s\_layer} \geq 0.003 b_e s_x = 0.67 \text{ in}^2 \end{cases}$

$s_x =$ 18.20 in

Crack spacing parameter (2) $$s_{xe} = s_x\frac{1.38}{a_g+0.63} =$$ 18.20 in (12.0 in ≤ s_{ex} ≤ 80.0 in)

Where, max aggregate size $a_g =$ 0.75 in

Shear resistance parameter $$\beta = \frac{4.8}{(1+750\varepsilon_s)}\frac{51}{(39+s_{xe})} =$$ 1.81 AASHTO 5.7.3.4.2

Concrete density modification factor $\lambda =$ 1.00 AASHTO 5.4.2.8

Nominal Shear Resistance $V_c = 0.0316\beta\lambda\sqrt{f'_c}\,b d_v =$ $0.0316\,(2)(1)\sqrt{4.50}\,(12)(18.20) =$ 26.50 kip

Factored Shear Resistance $V_R = \phi_v V_c =$ 0.90 (26.50) = 23.85 kip

Retaining wall footings and stems are typically unreinforced for shear. Confirm transverse reinforcement is not required by design, $0.5\,V_R > V_{u\,str}$ AASHTO 5.7.2.3

$0.5\,V_R =$ 11.93 kip

$0.5\,V_R > V_{u\,str}$ **OK**

2.1.3 Shrinkage and Temperature Reinforcement Design AASHTO 5.10.6

Horizontal reinforcement at each face of stem and vertical reinforcement at front face of stem

Try # 4 @ 12.0" on center: Design steel area $A_S =$ 0.200 in^2

Check $$A_s \geq \frac{1.30\,b\,T_{Bot}}{2(b+T_{Bot})f_y} =$$ 0.083 in^2 **OK**

Check $0.11 \leq A_s \leq 0.60$ **OK**

2.2 FOOTING HEEL DESIGN

The critical section for shear and moment is at the back face of the stem wall (C5.13.3.6). The heel is designed to carry its self weight and the soil block above it. Conservatively, it is common to ignore upward soil reaction under the footing heel, thus Strength 1b is not checked. For shear in footings, the provisions of 5.8.2.4 are not applicable, thus $\phi V_c \geq V_u$.

Summary of Unfactored Vertical Loads and Moments at the Back Face of the Stem:

Load Type	Description	V (kip/ft.)	Moment Arm (ft.)	M (kip-ft.)/ft.
DC	Heel dead load	1.03	2.75	2.83
EV_1	Vertical pressure from dead load of fill on heel	10.73	2.75	29.51

Summary of Load Groups:

Load Combination	Vertical Load & Moment	
	Vu (kip/ft.)	Mu (kip-ft.)/ft.
Strength IV	16.03	44.08
Service I	11.76	32.34

By inspection, load combination Strength IV generates a maximum moment at the interface of the footing heel and stem wall. However, the Designer should check all possible load combinations and select the combination that produces the maximum load for the design of the footing.

For reinforcement design, follow the procedure outlined in Section 2.1. Exposure Class I can be used for cracking check. Results of the design are as follows (also shown on Figure 3):

Transverse horizontal bar at top of footing - # 6 @ 6.0"

Longitudinal reinforcement, top and bottom of footing - # 4 @ 12.0"

2.3 FOOTING TOE DESIGN

The critical section for shear is d_V from front face of wall stem and, for moment, is at the front face of wall stem (C5.13.3.6). Section is designed to resist bearing stress acting on toe. This example conservatively ignores the soil on top of the toe. For shear in footings, the provisions of 5.8.2.4 are not applicable, thus $\phi Vc \geq Vu$.

Controlling loads:

Maximum bearing stress (factored)	σ_V =	3.08	ksf	(from bearing resistance check)
Factored shear	$V_{u\ str} = \sigma_V\ S$ =	8.47	kip/ft.	
Factored bending moment	$M_{u\ str} = V_u\ S/2$ =	11.65	kip-ft./ft.	

Service loads:

X =	$(\Sigma\ M_V - \Sigma\ M_H) / \Sigma\ V$ =	(130.18 - 33.30) / 20.53 =	4.72	ft.
e =	B / 2 - X =	10.0 / 2 - 4.72 =	0.28	ft.
σ_V =	ΣV / (B-2e) =	20.53 / (10.0 - 2 (0.28)) =	2.17	ksf

Factored shear	$V_{u\ serv} = \sigma_V\ S$ =	5.97	kip/ft.
Factored bending moment	$M_{u\ serv} = V_u\ S/2$ =	8.21	kip-ft./ft.

For reinforcement design, follow the procedure outlined in Section 2.1. Results of the design are as follows (also shown on Figure 3):

Transverse horizontal bar at bottom of toe - # 5 @ 6.0"

Note: Check that the toe length and footing depth can accommodate development length of the hooked bar past the design plane.

2.4 SHEAR KEY DESIGN

The critical section for shear and moment is at the interface with the bottom of the footing. Shear key reinforcing is designed to resist passive pressure determined in the sliding analysis. Passive pressure load resultant is located at a distance "z" from the bottom of footing, if using inclined wedge (see Figure 2).

Passive pressure against inert block R_{ep} = 8.00 kip

Moment arm $z = \left(0.5K_p\gamma_s y_1 c^2 + 0.333K_p\gamma_s c^3\right)/R_{ep} =$

$= [0.5\ (7.60)(0.130)(2.25)(2.36)^2 + 0.333\ (7.60)(0.130)(2.36)^3\ ]\ /\ 8.00 =$ 1.31 ft.

Factored bending moment for key design $M_{u\ str}$ = 10.48 kip-ft./ft.

For reinforcement design, follow the procedure outlined in Section 2.1. Results of the design are as follows (also shown on Figure 3):

Vertical 'U' bars at front and back face of shear key - # 4 @ 6.0"

Longitudinal reinforcement in shear key - # 4 @ 12.0"

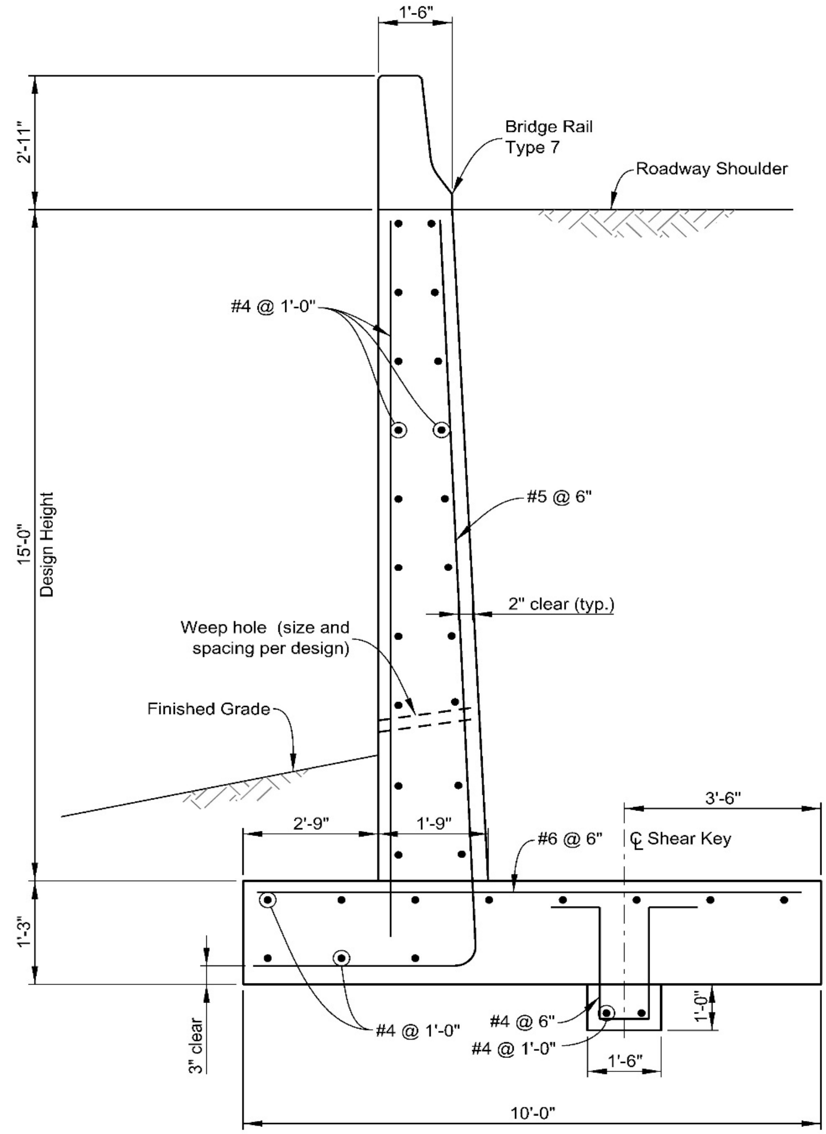

Figure 3 - Final Wall Section